The Glue Semantics Workbench

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About

- **GOAL:** Providing a modular, easy to use glue semantics tool written in Java that is useful for both computational linguists and formal semanticists
 - Provide a tractable, efficient implementation of (a fragment of) linear logic
 - Modular system that can be connected to various NLP pipelines, in particular XLE (Crouch et al., 2017), Stanford CoreNLP (Manning et al., 2014) with minimal effort
 - Illustration of the system by means of a classic formal semantic phenomenon
 - Free to use open-source software

Some existing resources:

- NLTK computational semantics package (Python)
- glue implementation PARC by Richard Crouch and colleagues (Prolog)

- "Instant Glue" prover by Miltiadis Kokkonidis (Prolog)
- glue prover algorithm outlined by (Lev, 2007)
- $\rightarrow\,$ served as initial guiding points

Why Java?

- object-oriented paradigm fits resource-sensitive nature of linear logic
- possibility to modularize the program
- many interfaces to libraries like the Stanford CoreNLP tools
- Java virtual machines ubiquitous across all operating systems
- Java is widely used both in academic and industrial software development

Background on linear logic

"[glue semantics] is an approach to the semantic interpretation of natural language that uses a fragment of linear logic as a deductive glue for combining together the meanings of words and phrases" -Crouch and van Genabith, (2000)

• linear logic (LL) is a resource-conscious logic premises, assumptions and conclusions as used in logical proofs are resources (not truths or facts)

$$\begin{array}{ll} A,A \rightarrow B,A \rightarrow C \models A,B,C \\ \text{vs. } A,A \multimap B,A \multimap C \not\models A,B,C \end{array} \\ \begin{array}{ll} \text{traditional} \\ \text{LL} \end{array}$$

 a sentence denotes a successful linear logic proof \rightarrow all resources introduced by the sentence have to be consumed

The appeal of linear logic

- syntax of proof systems of "traditional" logics is not always in one-to-one correspondence to the underlying proof object
- $\rightarrow\,$ LL better suited to describe underlying proof objects
 - resource usage occurs in natural language: Words and phrases correspond to resources
 - (Certain fragments) can be implemented in a tractable manner

Some technicalities

- lexical entries consist of two elements:
 - **glue language:** linear logic can be understood as semantic types (Curry-Howard-isomorphism)

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- meaning language Montague style semantics (but other formalism are possible)
- ex. $\lambda x.sleep(x) : A \multimap B$

ex. $\lambda P.\lambda Q.\exists x[P(x) \land Q(x)] : (A \multimap B) \multimap ((C \multimap D) \multimap D)$

Relevant rules

• we use the *implicational fragment* of linear logic

Introduction rule

Elimination rule

$$[x:A]^{i}$$

$$\vdots$$

$$\frac{f(x):B}{\lambda x.f(x):A \multimap B} \multimap_{I,i}$$

$$\frac{f:A\multimap B}{f(a):B} \xrightarrow{a:A} \multimap_E$$

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Semantic composition as proof

- John loves Mary.
- Iexical entries:

• $\llbracket Mary \rrbracket = m : h$

•
$$\llbracket \text{loves} \rrbracket = \lambda x . \lambda y . \text{loves}(x, y) : g \multimap (h \multimap f)$$

$$\frac{\lambda x.\lambda y.loves(x,y) : g \multimap (h \multimap f) \quad j : g}{\lambda y.loves(j,y) : h \multimap f} \qquad m : h$$
$$loves(j,m) : f$$

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From syntax to semantics

- $\begin{bmatrix} \mathsf{PRED 'love} < \mathsf{John}, \mathsf{Mary} >' \\ \mathsf{SUBJ} \begin{bmatrix} \mathsf{PRED 'John'} \\ \mathsf{OBJ} \end{bmatrix} \begin{bmatrix} \mathsf{PRED 'Mary'} \end{bmatrix}$ $\bullet \lambda x. \lambda y. \mathsf{loves}(x, y) : \\ \uparrow . \mathsf{SUBJ} \multimap (\uparrow . \mathsf{OBJ} \multimap \uparrow) \\ \bullet j : \uparrow . \mathsf{SUBJ}$ • *m* :↑ .*OBJ*
 - \uparrow refers to a specific f-structure node (e.g. \uparrow points to the outer f-structure; \uparrow .*SUBJ* points to the f-structure node of the subject)
 - syntactic analysis determines linear logic resources (see e.g. Dalrymple, 2001 and subsequent work)
 - traditionally co-descriptive, but description-by-analysis also possible (Kaplan, 1995)

Modules



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Prover algorithm

based on three principles, taken from algorithms by Hepple, (1996) and Gupta and Lamping, (1998)

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- I indexation
- II compilation
- III skeleton-modifier distinction

Hepple, (1996): Basic chart prover

Indexation

- Hepple parser stores partial results and re-uses them to prevent backtracking
 - linear use of resources enforced by using indexes
 - each LL formula (=premise) assigned unique index
 - when combining premises their index sets are unified
 - two premises can only be combined when their index sets are disjoint

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• Example:

j: g [0]

$$\lambda x.sleeps(x): g \multimap f$$
 [1]
 $sleeps(j): f$ [0,1]

first-order chart prover pseudo code

Stack A (agenda) List D (database) for A contains premises do pop premise P_A add P⊿ to D for all Premises P_D in D do if P_A and P_D combinable and index sets disjoint then add new combined premise to A end if end for end for if any P_D from D has a full set of indexes it is a valid solution

higher-order chart prover

Compilation

• higher-order formulas with nested consumers usually require ---o-introduction

- hypothetical reasoning makes computation very complex
- Hepple's solution: transform the initial (potentially higher-order) formulas into a set of first-order formulas
- nested consumers are "compiled out" to additional assumptions:

 $(a \multimap b) \multimap c [0] \Rightarrow$

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$$(a \multimap b) \multimap c [0] \Rightarrow b[a] \multimap c [0]$$

 $\{a\} [1]$

Higher-order chart prover

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Higher-order chart prover

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 - matched assumption and discharge pairs are removed from the book-keeping
- meaning side: compilation step amounts to functional application with deliberate "accidental binding" of relevant variable

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original premises:

 $\begin{array}{ll} g_1 \multimap f & : \lambda y.\mathsf{sleep}(\mathsf{y}) \\ (g_2 \multimap H) \multimap H & : \lambda \mathsf{P}.\forall \mathsf{x}[\mathsf{person}(\mathsf{x}) \land \mathsf{P}(\mathsf{x})] \end{array}$

compiled premises:

$g_1 \multimap f$: λ y.sleep(y)	
$\{g_2\}$: v	
$H[g_2] \multimap H$: $\lambda u.\lambda P.\forall x[person(x) \land P(x)](\lambda v.u)$	I)

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$$\{g_{2}\} \qquad : v$$

$$H[g_{2}] \multimap H \qquad : \lambda u.\lambda P.\forall x[\text{person}(x) \land P(x)](\lambda v.u)$$

$$\frac{g_{1} \multimap f : \lambda y.\text{sleep}(y) \quad \{g_{2}\} : v}{f\{g_{2}\} : \text{sleep}(v)}$$

$$\frac{f : \lambda P.\forall x[\text{person}(x) \land P(x)](\lambda v.\text{sleep}(v))}{f : \forall x[\text{person}(x) \land \text{sleep}(x)]} \beta \text{-conversion}$$

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Pseudo code: higher-order prover

```
Stack A (agenda)
List D (database)
Solutions S (all premises with full index sets)
for A contains premises do
               pop premise P_A
               add P_A to D
               for all Premises PD in D do
                              if P_A and P_D combinable and index sets disjoint then
                                             if P_A and/or P_D contain assumptions then
                                                            combine sets of assumptions
                                                            add new combined premise to A
                                             else if P_A or P_D contain discharges then
                                                            if discharges are a subset of assumptions then
                                                                           delete "used" discharges and assumptions
                                                                            add new combined premise to A
                                                            end if
                                             else
                                                            no assumptions or discharges; combine premises as usual
                                             end if
                              end if
               end for
end for
                                                                                                                                                                                                             < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □
```

Treating modifiers (following Gupta and Lamping, 1998)

skeleton-modifier distinction

- adjuncts like adjectives, adverbs, etc. significantly increase the complexity of a deduction
- need to be treated separately to prevent explosion of partial results
- separation between two types of glue premises:
 - modifier: each positive (producer) occurrence of a resource paired with negative (consumer) occurrence $(v_+ \multimap r_-)_- \multimap (v_- \multimap r_+)_+ : \lambda P.\lambda x.P(x) \land black(x)$
 - skeleton: premise with "unmatched" producer/consumer resources

 $(v_{-} \multimap r_{+}) : \lambda x.dog(x)$

- modifiers do not need to be compiled
- for each new skeleton premise taken from the agenda, check potential combination with modifiers

Syntax/semantics correspondence: quantifiers

Determiners

- the template for quantifiers is:
 (x → RESTR) → ((SCOPE →↑) →↑).
- the restrictor is always the dependency that governs the quantifier
- the scope is newly instantiated for a quantifier and later unified with the arguments of the verb.

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• g:
$$(x \multimap SUBJ) \multimap ((SCOPE_A \multimap \uparrow) \multimap \uparrow)$$

• h: $(x \multimap OBJ) \multimap ((SCOPE_B \multimap \uparrow) \multimap \uparrow)$
• $g \multimap (h \multimap f)$: $SCOPE_A \multimap (SCOPE_B \multimap \uparrow)$

Deriving ambiguities with the glue prover

An example with a quantifier scope ambiguity:

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Deriving ambiguities with the glue prover

After compilation we have the following premises:

(3) A dog chases every cat

$$\begin{array}{l} (g \multimap g) : \lambda y_e. dog(y) \quad [0] \\ (Y[h] \multimap (g[g] \multimap Y)) : \\ \lambda t_t. \lambda s_t. \lambda Q_{\langle e,t \rangle}. \lambda P_{\langle e,t \rangle}. \exists x [P(x) \land Q(x)](\lambda x''_e.s)(\lambda y''_e.t) \quad [1] \\ \{g\} : x'' \quad [5] \\ \{h\} : y'' \quad [6] \\ (i \multimap i) : \lambda x'_e. cat(x') \quad [2] \\ (X'[j] \multimap (i[i] \multimap X')) : \\ \lambda v_t. \lambda u_t. \lambda S_{\langle e,t \rangle}. \lambda R_{\langle e,t \rangle}. \forall z [R(z) \rightarrow S(z)](\lambda z''_e.u)(\lambda x''_e.v) \quad [3] \\ \{i\} : z'' \quad [7] \\ \{j\} : x''' \quad [8] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chases(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chases(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chases(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chases(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chases(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chases(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chases(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chases(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chases(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chases(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chases(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chases(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chases(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chases(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chase(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chase(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chase(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chase(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chase(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chase(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chase(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chase(y', z') \quad [4] \\ (h \multimap (j \lor f)) : \lambda y'_e. \lambda z'_e. chase(y', z') \quad [4] \\ (h \multimap (j \multimap f)) : \lambda y'_e. \lambda z'_e. chase(y', z') \quad [4] \\ (h \lor (j \lor f)) : \lambda y'_e. \lambda z'_e. chase(y', z') \quad [4] \\ (h \lor (j \lor f) : \lambda y'_e. \lambda z'_e. chase(y', z') \quad [4] \\ (h \lor (j \lor f) : \lambda y'_e. \lambda z'_e. chase(y', z') \quad [4] \\ (h \lor (j \lor f) : \lambda y'_e. \lambda z'_e. chase(y', z') \quad [4] \\ (h \lor (j \lor f) : \lambda y'_e. \lambda z'_e. chase(y', z') \quad [4] \\ (h \lor (j \lor f) : \lambda y'_e. \lambda z'_e. chase(y', z') \quad [4] \\ (h \lor (j \lor f) : \lambda y'_e. \lambda z'_e. chase$$

Deriving ambiguities with the glue prover

$$\frac{(h \multimap (j \multimap f)) : \lambda y'_e . \lambda z'_e . chases(y', z')[4] \qquad \{h\} : y''[6]}{(j \multimap f)\{h\} : \lambda z'_e . chases(y'', z')[4, 6]} \qquad \{j\} : x'''[8]}{f\{j, h\} : chases(y'', x''')[4, 6, 8]}$$

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Deriving ambiguities with the glue prover

$$\frac{(h \multimap (j \multimap f)) : \lambda y'_e . \lambda z'_e . chases(y', z')[4] \qquad \{h\} : y''[6]}{(j \multimap f)\{h\} : \lambda z'_e . chases(y'', z')[4, 6]} \qquad \{j\} : x'''[8]}{f\{j, h\} : chases(y'', x''')[4, 6, 8]}$$

surface scope reading:

$$\frac{f\{j,h\}[4,6,8] \qquad (X'[j] \multimap (i[i] \multimap X'))[3]}{(i[i] \multimap f)\{h\}[3,4,6,8]}$$

$$\frac{f\{h\}[2,3,4,6,7,8] \qquad (Y[h] \multimap (g[g] \multimap Y))[1]}{(g[g] \multimap f)[1,2,3,4,6,7,8]} \qquad g\{g\}[0,5]$$

$$\frac{f\{h\}[2,3,4,6,7,8] \qquad (Y[h] \multimap (g[g] \multimap Y))[1]}{f: \exists x[dog(x) \land \forall z[cat(z) \to chases(x,z)]][0,1,2,3,4,5,6,7,8]}$$

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Deriving ambiguities with the glue prover

$$\frac{(h \multimap (j \multimap f)) : \lambda y'_e . \lambda z'_e . chases(y', z')[4] \qquad \{h\} : y''[6]}{(j \multimap f)\{h\} : \lambda z'_e . chases(y'', z')[4, 6]} \qquad \{j\} : x'''[8]}{f\{j, h\} : chases(y'', x''')[4, 6, 8]}$$

surface scope reading:

$$\frac{f\{j,h\}[4,6,8] \qquad (X'[j] \multimap (i[i] \multimap X'))[3]}{(i[i] \multimap f)\{h\}[3,4,6,8]}$$

$$\frac{f\{h\}[2,3,4,6,7,8] \qquad (Y[h] \multimap (g[g] \multimap Y))[1]}{(g[g] \multimap f)[1,2,3,4,6,7,8]} \qquad g\{g\}[0,5]$$

$$\overline{f: \exists x[dog(x) \land \forall z[cat(z) \to chases(x,z)]][0,1,2,3,4,5,6,7,8]}$$

inverse scope reading:

$$\frac{f\{j,h\}[4,6,8] \quad (Y[h] \multimap (g[g] \multimap Y))[1]}{g\{g\}[0,5] \quad (g[g] \multimap f)\{j\}[1,4,6,8]}$$

$$\frac{f\{j\}[0,1,4,5,6,8] \quad (X'[j] \multimap (i[i] \multimap X'))[3]}{(i[i] \multimap f)[0,1,3,4,5,6,8]} \quad i\{i\}[2,7]$$

$$\frac{f\{j\}[0,1,4,5,6,8] \quad (X'[j] \multimap (i[i] \multimap X'))[3]}{f: \forall z[cat(z) \to \exists x[dog(x) \land chasse(x,z)]][0,1,2,3,4,5,6,7,8]}$$

23/29

The Glue Semantics Workbench in action

```
Selected file chase webXLE.pl
[(g \Rightarrow g) : \lambda x \text{ e.cat}(x)[0], ((g \Rightarrow g) \Rightarrow ((h \Rightarrow Y) \Rightarrow Y)) : \lambda P < e, t > \lambda Q < e, t > . \forall y [P(y) \rightarrow Q(y)][1], (i \Rightarrow i) : \lambda z \text{ e.dog}(z)[2], (i \Rightarrow i) : (i \Rightarrow i) : \lambda z \text{ e.dog}(z)[2], (i
Searching for valid proofs ...
Agenda: [(g → g) : λx e.cat(x)[0], {g} : x''[5], {h} : y''[6], (Y[h] → (g[g] → Y)) : λt t.λs t.λQ <e,t>.λP <e,t>.∀y[P(y]
Combining premises \{g\} : x''[5] and (g \neg g) : \lambda x = .cat(x)[0]
--> g{g} : cat(x'')[0, 5]
Combining premises {i} : z''[7] and (i \neg i) : \lambda z = .dog(z)[2]
--> i{i} : dog(z'')[2, 7]
Combining premises (j \rightarrow (h \rightarrow f)) : \lambda y' = \lambda z' = chase(y', z')[4] and \{j\} : x'''[8]
-->(h \rightarrow f)\{j\} : \lambda z' = .chase(x''', z')[4, 8]
Combining premises (h = f){j} : \lambda z' e.chase(x''',z')[4, 8] and {h} : y''[6]
-->f{j,h} : chase(x''',y'')[4, 6, 8]
Combining premises f{j,h} : chase(x''',y'')[4, 6, 8] and (X'[j] = (i[i] = X')) : hv t.hu t.hS <e,t>.AR <e,t>.AR <e,t>.Ar <e,t>.
--> (i[i] → f){h} : λu t.λS <e,t>.λR <e,t>.∃x'[R(x') ∧ S(x')](λz''_e.u)(λx'''_e.chase(x''',y''))[3, 4, 6, 8]
Combining premises f{j,h} : chase (x''', y'') [4, 6, 8] and (Y[h] \Rightarrow (g[q] \Rightarrow Y)) : \lambda t \ t.\lambda s \ t.\lambda Q \ (e, t>.\lambda P \ (e, t>.\forall y[P(y) \Rightarrow Q(t)))
--> (g[g] → f){j} : λs t.λQ <e,t>.λP <e,t>.∀y[P(y) → Q(y)](λx'' e.s)(λy'' e.chase(x''',y''))[1, 4, 6, 8]
Combining premises (q[q] \Rightarrow f); \lambda = t, \lambda < (e, t), 
-->f{i}: \forall v[cat(v) \rightarrow chase(x'',v)][0, 1, 4, 5, 6, 8]
Combining premises f(j) : \forall y[cat(y) \rightarrow chase(x'',y)][0, 1, 4, 5, 6, 8] and (X'[j] \neg (i[i] \neg X')) : \lambda y t.\lambda u t.\lambda s <e,t>.\lambda
--> (i[i] → f) : Au t.As <e,t>.Ar <e,t>.∃x'[R(x') ∧ s(x')](Az'' e.u)(Ax''' e.∀y[cat(y) → chase(x''',y)])[0, 1, 3, 4, 5,
 \text{Combining premises (i[i] < f): } \lambdau_t.\lambdas_{e,t>.\lambdaR_{e,t>.}X'[R(x') \land s(x')](\lambda z''_e.u)(\lambda x'''_e.\forall y[cat(y) \rightarrow chase(x''',y)]) } 
-->f : ∃x'[dog(x') ∧ ∀y[cat(y) → chase(x',y)]][0, 1, 2, 3, 4, 5, 6, 7, 8]
Combining premises (i[i] ⊲ f){h} : λu_t.λS_<e,t>.λR_<e,t>.∃x'[R(x') ∧ S(x')](λz''_e.u)(λx'''_e.chase(x''',y''))[3, 4, 6,
-->f{h} : ∃x'[dog(x') ∧ chase(x',y'')][2, 3, 4, 6, 7, 8]
Combining premises f{h} : ∃x'[dog(x') ∧ chase(x',y'')][2, 3, 4, 6, 7, 8] and (Y[h] → (g[g] → Y)) : λt t.λs t.λQ <e,t>.λI
--> (g[g] ⊲ f) : As t.Ag <e,t>.AP <e,t>.∀y[P(y) → g(y)](Ax'' e.s)(Ay'' e.∃x'[dog(x') ∧ chase(x',y'')])[1, 2, 3, 4, 6, 7]
Combining premises (g[g] \neg f) : \lambda s t.\lambda Q < e, t > \lambda P < e, t > \forall y [P(y) \rightarrow Q(y)] (\lambda x'' e.s) (\lambda y'' e.\exists x'[dog(x') \land chase(x', y'')]) [1]
-->f : ∀y[cat(y) → ∃x'[dog(x') ∧ chase(x',y)]][0, 1, 2, 3, 4, 5, 6, 7, 8]
Found valid deduction(s):
f : \exists x' [dog(x') \land \forall y [cat(y) \rightarrow chase(x', y)] ] [0, 1, 2, 3, 4, 5, 6, 7, 8]
f : \forall v [cat(v) \rightarrow \exists x' [dog(x') \land chase(x',v)]][0, 1, 2, 3, 4, 5, 6, 7, 8]
Done!
```

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From f-structures to glue premises

```
cf(1,eq(attr(var(0), 'PRED'),semform('eat',2,[var(10),var(2)],[])))
cf(1,eq(attr(var(0), 'SUBJ'),var(10))),
cf(1,eq(attr(var(0), 'OBJ'),var(2))),
...
cf(1,eq(attr(var(10), 'PRED'),var(14))),
cf(1,eq(var(14),semform('Pluto',0,[],[]))),
...
cf(1,eq(attr(var(2), 'PRED'),semform('bone',6,[],[]))),
```

- pattern-based parser extracts all grammatical functions and their PRED-values
- The system first generates lexical entries for grammatical functions and then generates the verbal spine
- \rightarrow Description-by-analysis
- $\rightarrow\,$ May be outsourced to XLE transfer system

From dependencies to glue premises



- in LFG we make use of the flat f-structure to determine relations between syntax and semantics
- $\rightarrow\,$ We can simply flatten the dependency structure into a list of dependency facts using the underlying similarities of the two formalisms

Summary

- We presented a semantic parser at the core of which is a chart prover for linear logic formulas that decomposes higher order linear logic formulas into first order formulas.
- We implemented corresponding semantics that can be applied to natural language.
- We provide a small system for translating dependency parses and Prolog f-structure files into default semantic premises that can be proven/composed with the parser.
- The program can be easily extended/modified:
 - lexicon: implementing a proper, potentially co-descriptive lexicon
 - semantics: hook up with various semantic formalisms (e.g. DRT)

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